Chiral-odd Fragmentation Functions in Single Pion Inclusive Electroproduction

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Abstract

We consider a sub-leading twist chiral-odd pion fragmentation function and explore its contribution in single pion semi-inclusive electroproduction. We evaluate the single beam-spin azimuthal asymmetry A_{LU} and the double spin asymmetry A_{LT} in polarized electroproduction of pions from an unpolarized and transversely polarized nucleon respectively. The beam asymmetry is expressed as the product of chiral-odd, and T-odd and even distribution and fragmentation functions. The double spin asymmetry contains information on the quark's transversity distribution. In a quark diquark-spectator framework we estimate these asymmetries at 6 GeV, 12 GeV, and 27.5 GeV energies.

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1 Introduction

One of the most interesting results in deep-inelastic spin physics has been the discovery of a class of chirally odd quark distribution functions. That which has garnered most attention is the leading twist transversity distribution h_1 which provides information on the quark transverse spin distribution in a transversely polarized nucleon [1,2]. Chiral-odd distribution functions are difficult to measure because they are suppressed in inclusive deep inelastic scattering. However, when two hadrons participate in the scattering process, the nucleon's transversity can be accessed; for example, in Drell-Yan scattering with transversely polarized protons [1,3]. Alternatively, transversity can be probed in semi-inclusive deep inelastic scattering (SIDIS) where outgoing hadrons are produced in the current fragmentation region. This process [4] has been used as a filter to access transversity [5]. Here,

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the probability for a transversely polarized quark to produce a pion is probed. Other methods to probe transversity involving semi-inclusive production of Λ hyperons, and of two pions have also been discussed in the literature [6,7,8].

Some time ago, Jaffe and Ji [9] suggested that the nucleon's transversity could be probed in polarized electroproduction of pions from a transversely polarized nucleon. By comparison with the above mentioned Drell-Yan and single-spin asymmetry approaches, their proposal is sub-leading in twist. However, since the measurement involves merely one spinless particle in the final state it proves to be an interesting approach to probe the effects of higher twist in addition to providing a window into the measurement of transversity. The asymmetry characterizing this process consists of a linear combination of two terms, one chiral-even and one chiral-odd. To expose the the chiral-odd effect of interest [10], the competing chiral-even mechanism must be subtracted away.

In this letter we will estimate the relative magnitudes of these two contributions to the double spin asymmetry in the quark-diquark spectator framework. In doing so, we explore the sub-leading twist chirally odd pion fragmentation function E (in the Ref. [9] it was denoted by $\hat{e}_1$). On the other hand, as it will be shown below, that chiral-odd fragmentation function can show up also in the SIDIS beam spin asymmetry (BSA) in addition to effects considered by Levelt and Mulders [11]. The interest in the BSA for pion electroproduction in semi-inclusive deep inelastic scattering of longitudinally polarized electrons off unpolarized nucleon resides in the fact that the beam probes the antisymmetric part of the hadron tensor, which is particularly sensitive to final state interactions. In longitudinally polarized electron electromagnetic scattering, the BSA shows up as a $\langle \sin\phi \rangle$ asymmetry for the produced hadron and is expressed as

$$< \sin\phi > = \pm \langle \frac{\mathsf{s} \times \boldsymbol{k}_2 \cdot \boldsymbol{P}_{h\perp}}{|\mathsf{s} \times \boldsymbol{k}_2||\boldsymbol{P}_{h\perp}|} \rangle, \tag{1}$$

where s denotes the spin vector of the electron (the upper (lower) sign for right (left) handed electrons), $\boldsymbol{k}_1$ ($\boldsymbol{k}_2$) is three-dimensional vector of incoming (outgoing) electron momentum and $\boldsymbol{P}_{h\perp}$ is the produced hadron's transverse momentum about virtual photon direction; ϕ is the azimuthal angle of produced pions relative to the lepton scattering plane, and ϕ_S is the azimuthal angle of the target polarization vector (Fig. 1). This asymmetry is related to the left-right asymmetry in the hadron momentum distribution with respect to the electron scattering plane,

$$A = \frac{\int_0^\pi d\phi d\sigma - \int_\pi^{2\pi} d\phi d\sigma}{\int_0^\pi d\phi d\sigma + \int_\pi^{2\pi} d\phi d\sigma}, \tag{2}$$

which is $4/\pi$ times $< \sin\phi >$. Here $d\sigma$ is a shorthand notation for $d\sigma^{\vec{e}N\to ehX}/dx\,dy\,dz\,d^2P_{h\perp}$, and x, y, and z are the standard leptoproduction scaling variables [11]. As will be shown in this letter, the asymmetry is a superposition of $e \star H_1^\perp$, obtained in Ref. [11], and $h_1^\perp \star E$. A similar result has been presented very recently by Yuan [12], where the BSA has been phenomenologically studied in the extrema case of $h_1^\perp \star E$. By contrast the BSA has been studied at the other extreme, $e \star H_1^\perp$ [13,14] while in addition the bounds on this asymmetry were considered in [15]. Further, the BSA has been considered solely from perturbative QCD effects at second order in α_S [16,17,18]. In the Ref. [19] a dynamical model for BSA similar to that used for the single target spin asymmetries [20] has been proposed.

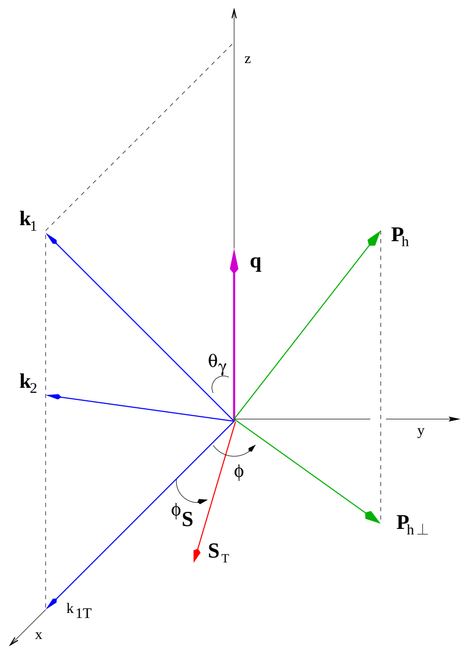

Fig. 1. The kinematics of SIDIS.

This letter is organized as follows: First we calculate the sub-leading twist chiral-odd fragmentation function E, defined by Jaffe and Ji [9], following the approach in [21,22]. Then we study its physical implications in filtering the nucleon's transversity properties by considering the double-spin asymmetry, A_{LT}, in polarized electroproduction of pions from a transversely polarized nucleon. This process contains information on quark's transversity distribution. We also consider the single beam-spin azimuthal asymmetry, A_{LU}. We perform order of magnitude estimates of these asymmetries for HERMES and ongoing and upgraded JLAB energies [5,23]. With regard to A_{LT}, we estimate that the chiral-odd effect is small and apparently its isolation from the chiral-even contribution would appear to be challenging measurement.

2 The A_{LT} and A_{LU} Asymmetries in the Spectator Framework

Here we focus on the the chirally odd transverse momentum dependent distribution and fragmentation functions, $e(x,\boldsymbol{p}_T)$ and $E(z,-z\boldsymbol{k}_T)$. As mentioned in the introduction, the fragmentation function $E(z,-z\boldsymbol{k}_T)$ arises when a longitudinal polarized electron beam probes a transversely polarized nucleon. Quantitatively this is represented in the joint product $h_1(x)\star E(z)$ that arises in the asymmetry A_{LT} which is accompanied with the more commonly investigated chiral-even combination, $g_T(x)\star D_1(z)$. On the other hand, the chirally odd distribution function $e(x)$ contributes in the combination $e(x)\star H_1^\perp(z)$ in the beam asymmetry, where $H_1^\perp(z)$ correlates the probability for a transversely polarized quark to fragment to a pion. This term is accompanied with the complement T-odd combination $h_1^\perp(x)\star E(z)$. The latter combination provides an additional term that fuels the beam asymmetry. The function $E(z,-z\boldsymbol{k}_T)$ is projected from the fragmentation matrix $\Delta(k,P_h)$ using the identity operator

$$\Delta^{[\mathbf{1}]}(z,\boldsymbol{k}_T)=\frac{1}{4z}\int dk^+\mathrm{Tr}\left(\mathbf{1}\Delta(k,P_h)\right)\Big|_{k^-=\frac{P_\pi^-}{z},\boldsymbol{k}_T}=\frac{M_h}{P_\pi^-}E(z,z^2\boldsymbol{k}_T^2), \tag{3}$$

where $\Delta(z,\ \boldsymbol{k}_T)$ is parameterized in terms of the relevant fragmentation functions

$$\Delta(z,\boldsymbol{k}_T) = \frac{1}{4}\Big\{D_1(z,-z\boldsymbol{k}_T)\not{n}_- + H_1^{\perp}(z,-z\boldsymbol{k}_T)\frac{\sigma^{\alpha\beta}k_{T\alpha}n_{-\beta}}{M_h} + \frac{M_h}{P_h^-}E(z,-z\boldsymbol{k}_T) + \cdots\Big\}. \tag{4}$$

Similarly, $e(x,\boldsymbol{p}_T)$ is projected from the distribution matrix $\Phi(p,P)$,

$$\Phi^{[\mathbf{1}]}(x,\boldsymbol{p}_T) = \frac{1}{2}\int dk^- \mathrm{Tr}\left(\mathbf{1}\Phi(p,P)\right)\Big|_{p^+=xP^+,\boldsymbol{p}_T} = \frac{M}{P^+}e(x,\boldsymbol{p}_T^2) \tag{5}$$

which is parameterized as

$$\begin{aligned}\Phi(x,\boldsymbol{p}_T) = \frac{1}{2}\Big\{&f_1(x,\boldsymbol{p}_T)\not{n}_+ + h_1^{\perp}(x,\boldsymbol{p}_T)\sigma^{\alpha\beta}p_{T\alpha}n_{+\beta}M + h_{1T}(x,\boldsymbol{p}_T)i\gamma_5\sigma^{\alpha\beta}n_{+\alpha}S_{T\beta} \\ &+ h_{1s}^{\perp}(x,\boldsymbol{p}_T)\frac{i\gamma_5\sigma^{\alpha\beta}n_{+\alpha}p_{T\beta}}{M} + \frac{M}{P^+}\Big[e(x,\boldsymbol{p}_T) + g_T'(x,\boldsymbol{p}_T)\gamma_5\not{S}_T \\ &+ g_s^{\perp}(x,\boldsymbol{p}_T)\frac{\gamma_5\not{p}_T}{M}\Big]\cdots\Big\}\,,\end{aligned} \tag{6}$$

where we have used the shorthand naming convention [24,25]

$$h_{1s}^{\perp}(x,\boldsymbol{p}_T) \equiv \lambda h_{1L}^{\perp}(x,\boldsymbol{p}_T) + \frac{(\boldsymbol{p}_T\cdot S_T)}{M}h_{1T}^{\perp}(x,\boldsymbol{p}_T). \tag{7}$$

We calculate these functions in the spectator model framework [26,27]. To address the log divergence [26,20,28,29,30,31] that arises when calculating the moments of distribution and fragmentation functions that appear in asymmetries, we introduce a Gaussian distribution in the transverse momentum dependence of the quark-spectator-pion and quark-nucleon-spectator vertices [26,21,22]. This serves to smoothly cutoff the integration in $\boldsymbol{k}_T$ which kinematically parameterizes our knowledge of confining effects. For the fragmentation vertex we couple the on-shell spectator, as a quark interacting with the produced pion (hereafter, $P_h = P_\pi$) through the vertex function

$$\langle 0|\psi(0)|P;X\rangle = \left(\frac{i}{\not{k}-m}\right)\Upsilon(\boldsymbol{k}_T^2)\,U(k-P_\pi,s), \quad \text{where} \quad \Upsilon(\boldsymbol{k}_T^2) = i\gamma_5 f_{qq\pi}e^{-b'\boldsymbol{k}_T^2}\,. \tag{8}$$

Here, $f_{qq\pi}(\equiv f)$ is the quark-pion coupling and k is the momentum of the off-shell quark, $\boldsymbol{k}_T$ and $b' = 1/<\boldsymbol{k}_T^2>$, are the intrinsic transverse momentum and its inverse mean square respectively, and $U(p,s)$ is the off-shell quark spinor. A similar analysis applies to the quark-nucleon-spectator vertex as it relates to the distribution function. Using Eqs. (3,8), the $\boldsymbol{k}_T$ integrated chiral-odd twist-three fragmentation function is $E(z)$

$$E(z) = \frac{m}{P^-}\frac{f^2}{4(2\pi)^2}\frac{1}{z}\frac{(1-z)^2}{z^2}\left\{\frac{m_\pi^2}{\Lambda'(0)} - 2b'm_\pi^2 e^{2b'\Lambda'(0)}\Gamma(0,2b'\Lambda'(0))\right\}, \tag{9}$$

where $\Lambda'(0) = \frac{1-z}{z^2}M_\pi^2 + \frac{\mu^2}{z} - \frac{1-z}{z}m^2$. The T-even distribution functions $f_1(x)$, $h_1(x)$, and fragmentation functions, $D_1(z)$, $H_1^{\perp}(z)$ are detailed in [21] and [22] (see also [33]). Similarly using Eq. (5), the $\boldsymbol{p}_T$ integrated chiral-odd distribution $e(x)$ function is

$$
\begin{aligned}
e(x) = & \frac{M}{4P^+}\frac{g^2}{(2\pi)^2}\Bigg\{\frac{(1-x)\,(m+xM)\,(m+M)-m^2\left(x+\frac{m}{M}\right)+\Lambda(0)\left(1+\frac{m}{M}\right)}{\Lambda(0)} \\
& -\Bigg[2b\Bigg(\,(1-x)\,(m+xM)\,(m+M)-m^2\left(x+\frac{m}{M}\right)+\Lambda(0)\left(1+\frac{m}{M}\right)\Bigg) \\
& \qquad\qquad +\left(1+\frac{m}{M}\right)\Bigg]\times\; e^{2b\Lambda(0)}\Gamma(0,2b\Lambda(0))\Bigg\}\,, \qquad (10)
\end{aligned}
$$

where g is the scalar diquark coupling [27], $\Lambda(0)=(1-x)m^2+x\lambda^2-x(1-x)M^2$, while M and m are the nucleon and quark masses respectively. Choosing $<p_T^2>=(0.4)^2\ \mathrm{GeV}^2=1/b$, yields good agreement [21,22] between $f_1(x)$ and the corresponding valence distribution of Ref. [32]. Additionally the chiral-even polarized function is projected from Eq. (6),

$$
\begin{aligned}
g_T(x) = & \frac{M}{4P^+}\frac{g^2}{(2\pi)^2}\Bigg\{\frac{(1-x)\,(m+xM)\,(m+M)-(m^2-\Lambda(0))\left(x+\frac{m}{M}\right)}{\Lambda(0)} \\
& -\Bigg[2b\left((1-x)\,(m+xM)\,(m+M)-\left(m^2-\Lambda(0)\right)\left(x+\frac{m}{M}\right)\right)+\left(x+\frac{m}{M}\right)\Bigg] \\
& \qquad\qquad \times e^{2b\Lambda(0)}\Gamma(0,2b\Lambda(0))\Bigg\}\,. \qquad (11)
\end{aligned}
$$

The distribution and fragmentation functions enter cross section for one-particle inclusive deep inelastic scattering which is given by

$$
\frac{d\sigma^{\ell+N\to\ell'+h+X}}{dx\,dy\,dz\,d^2P_{h\perp}} = \frac{\pi\alpha^2 y}{2Q^4 z}L_{\mu\nu}2M\mathcal{W}^{\mu\nu} = \frac{2\,\pi\alpha^2}{Q^2 y}\sum_a e_a^2\sigma^a, \qquad (12)
$$

where the factorized [24] hadronic tensor is

$$
\begin{aligned}
2M\mathcal{W}^{\mu\nu}(q,P,P_h) = & \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\ \delta^2(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T) \\
& \times\frac{1}{4}\ \mathrm{Tr}\Big[\Phi(x_B,\boldsymbol{p}_T)\gamma^\mu\Delta(z_h,\boldsymbol{k}_T)\gamma^\nu\Big]\ +\ \Big(q\leftrightarrow -q\ ,\ \mu\leftrightarrow\nu\Big)\ , \qquad (13)
\end{aligned}
$$

and $L_{\mu\nu}$ is the well-known lepton tensor. To investigate the $\sin\phi$ BSA and ϕ-independent σ_{LT} SIDIS cross section we keep only those terms producing contributions to Eq. (13) [1]

$$
\begin{aligned}
2M\,\mathcal{W}^{\mu\nu} = & 2z\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\delta^2(\boldsymbol{p}_T-\frac{\boldsymbol{P}_{h\perp}}{\boldsymbol{z}}-\boldsymbol{k}_T)\times\Bigg\{-g_\perp^{\mu\nu}f_1D_1+i\,\frac{2\,\hat{t}^{\{\mu}k_T^{\nu\}}}{Q}\frac{M}{M_h}\,x\,e\,H_1^\perp \\
& -i\,\frac{2\,\hat{t}^{\{\mu}p_T^{\nu\}}}{Q}\frac{M_h}{M}\,h_1^\perp\,\frac{E}{z}+\,i\,\frac{2\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}p_{T\rho}}{Q}\left[\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M}\left(x\,g_T^\perp\,D_1\,+\frac{M_h}{M}h_{1T}^\perp\,\frac{E}{z}\right)\right] \\
& +i\frac{2M\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\,\rho}\,S_{T\rho}}{Q}\left[x\,\left(g_T-\frac{\boldsymbol{p}_T^2}{2M^2}\,g_T^\perp\right)\,D_1\,+\frac{M_h}{M}\,\frac{E}{z}\,\left(h_1\,-\frac{\boldsymbol{p}_T^2}{2M^2}\,h_{1T}^\perp\right)\right]\Bigg\}. \qquad (14)
\end{aligned}
$$

[1] To avoid ambiguities, we will use the same notations as in Ref. [24]. Also the terms proportional to the current quark mass are neglected.

Contracting the hadronic tensor with the helicity dependent part of the leptonic tensor leads to the reduced cross sections which contribute to Eq. (12)

$$
\begin{aligned}
\sigma^a = \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, z^2 \delta^2(\boldsymbol{P}_{h\perp} - z(\boldsymbol{p}_T - \boldsymbol{k}_T)) \times \Bigg\{ & \left[1 + (1-y)^2\right]\; f_1^a(x, \boldsymbol{p}_T^2) D_1^a(z,\, z^2\boldsymbol{k}_T^2) \\
& - 4\lambda_e\, y \sqrt{1-y}\, \frac{1}{Q}\, \frac{M}{M_h} k_{Ty}\, x\, e^a(x,\, \boldsymbol{p}_T^2) H_1^{\perp\, a}(z,\, z^2\boldsymbol{k}_T^2) \\
& + 4\lambda_e\, y \sqrt{1-y}\, \frac{1}{Q}\, \frac{M_h}{M} p_{Ty}\, h_1^{\perp\, a}(x,\, \boldsymbol{p}_T^2) \frac{E^a(z,\, z^2\boldsymbol{k}_T^2)}{z} \\
& + 4\lambda_e\, y \sqrt{1-y}\, \frac{1}{Q}\, S_{Tx} \left(M\, x\, g_T^a(x,\, \boldsymbol{p}_T^2) D_1^a(z,\, z^2\boldsymbol{k}_T^2) + M_h\, h_1^a(x,\, \boldsymbol{p}_T^2) \frac{E^a(z,\, z^2\boldsymbol{k}_T^2)}{z} \right) \Bigg\}.
\end{aligned}
\tag{15}
$$

Here, k_{Ty} (p_{Ty}) denote the y component of the final (initial) parton transverse momentum vector and S_{Tx} denotes the x component of the nucleon's polarization vector. We project the weighted differential cross section integrated over the transverse momentum of the produced hadron [34,25]

$$
\langle W \rangle_{AB} = \int d^2 P_{h\perp}\, W \frac{d\sigma^{\ell + N \to \ell' + h + X}}{dx dy dz d^2 P_{h\perp}}, \tag{16}
$$

from Eq. (15) where $W = W(P_{h\perp}, \phi, \phi_S)$. The subscripts AB represent the polarization of lepton and target hadron respectively, U for unpolarized, L for longitudinally polarized and T for transversely polarized particles. From Eq. (12) the relevant reduced cross sections terms are [2]

$$
\sigma_{UU} \equiv \langle 1 \rangle_{UU} = \frac{[1 + (1-y)^2]}{y}\, f_1(x) D_1(z), \tag{17}
$$

$$
\sigma_{LT} \equiv \langle 1 \rangle_{LT} = \lambda_e |\boldsymbol{S}_T| \sqrt{1-y}\, \frac{4}{Q}\, \cos\phi_S \left[M\, x\, g_T(x) D_1(z) + M_h\, h_1(x) \frac{E(z)}{z} \right], \tag{18}
$$

$$
\langle |P_{h\perp}| \sin\phi \rangle_{LU} = \lambda_e \sqrt{1-y}\, \frac{4}{Q}\, M M_h \left[x\, e(x) z H_1^{\perp\,(1)}(z) \; + h_1^{\perp\,(1)}(x)\, E(z) \right], \tag{19}
$$

where $h_1(x) = h_{1T}(x) + h_{1T}^{\perp(1)}(x)$, and the weighted cross section contain the $\boldsymbol{p}_T^2$- and $\boldsymbol{k}_T^2$-moments of the distribution and fragmentation functions,

$$
h_1^{\perp(1)}(x) \equiv \int d^2\boldsymbol{p}_T \frac{\boldsymbol{p}_T^2}{2M^2}\, h_1^{\perp}(x, \boldsymbol{p}_T^2), \;\text{ and }\; H_1^{\perp(1)}(z) \equiv z^2 \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2M_h^2} H_1^{\perp}(z, z^2\boldsymbol{k}_T^2). \tag{20}
$$

[2] Hereafter we omit a assuming that the cross section is given predominantly by scattering on the u-quark.

In turn, the asymmetries for which we will give an estimate are the weighted integrals of a SIDIS cross section, Eq. (16):

$$A_{LU} \equiv A_{LU}^{|P_{h\perp}|\sin\phi} \equiv \frac{\int d^2P_{h\perp}|P_{h\perp}|\,\sin\phi\,(\sigma^{\leftarrow}-\sigma^{\rightarrow})}{\frac{1}{2}\int d^2P_{h\perp}\,(\sigma^{\leftarrow}+\sigma^{\rightarrow})} = 2\,\frac{\langle|P_{h\perp}|\sin\phi\rangle_{LU}}{\sigma_{UU}}, \tag{21}$$

$$A_{LT} \equiv \frac{\int d^2P_{h\perp}\,(\sigma^{\leftarrow}(\phi_S)+\sigma^{\rightarrow}(\pi+\phi_S)-\sigma^{\leftarrow}(\phi_S)-\sigma^{\rightarrow}(\pi+\phi_S))}{\int d^2P_{h\perp}\,(\sigma^{\leftarrow}(\phi_S)+\sigma^{\rightarrow}(\pi+\phi_S)+\sigma^{\leftarrow}(\phi_S)+\sigma^{\rightarrow}(\pi+\phi_S))} = \frac{\sigma_{LT}}{\sigma_{UU}}. \tag{22}$$

Here $\sigma^{\leftarrow}(\phi_S)$, $(\sigma^{\rightarrow}(\pi+\phi_S))$ denote the cross section with anti-parallel (parallel) polarization of the beam and for a transversely polarized target. In numerical calculations we assume 100% beam and target polarization and $\cos\phi_S = 1$.

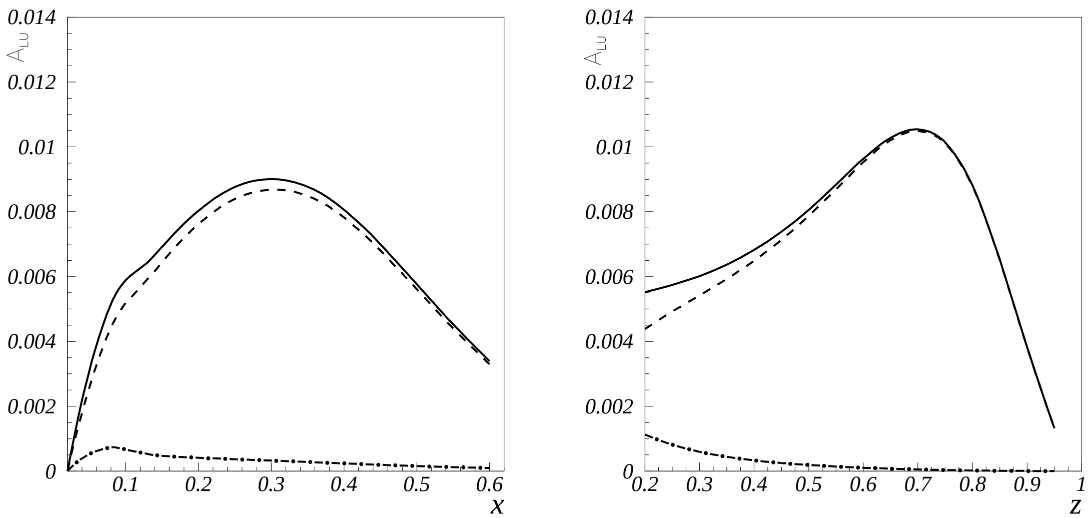

Fig. 2. A_{LU} for π^+ production as a function of x and z at 27.5 GeV energy. The dashed and dot-dashed curves correspond to contribution of the first and second terms of Eq. (19) respectively, and the full curve is the sum of the two.

The curves in Figs. 2, 3, and 4 are calculated at 27.5 GeV, 12 GeV, and 6 GeV beam energies by integrating over the kinematic ranges corresponding to $0.1 \le y \le 0.85$, $Q^2 \ge 1$ GeV^2, and $E_\pi \ge 2.0$ GeV. In Fig. 2, the asymmetry A_{LU} of Eq. (21) for π^+ production on a proton target is presented as a function of x and z. The dashed and dot-dashed curves correspond to contribution of the two terms of Eq. (19) respectively, and the full curve is the sum of the two. From Fig. 2 one can see that the contribution of the second term of Eq. (21), $h_1^{\perp(1)}(x)\,E(z)$, to the beam spin asymmetry is negligible whereas the first term, $e(x)H_1^{\perp(1)}(z)$, dominates. This is to be contrasted with the result obtained in the Ref. [12], where the z dependence of the $A_{LU}^{\sin\phi}$ results solely from the ratio of $E(z)$ to $D_1(z)$ calculated in the chiral quark model [35]. In Fig. 3, the BSA, A_{LU}, is presented. The dashed curve corresponds to the full asymmetry at 6 GeV beam energy and similarly, the full curve corresponds to 12 GeV beam energy. It is apparent that decreasing the beam energy results in an increasing BSA, which is consistent with it being a twist-three effect, suppressed by $\mathcal{O}(1/Q)$. In Fig. 4, the asymmetry $A_{LT}(x)$ of Eq. (22) for π^+ production as a function of Bjorken x and z is presented. The dashed and dot-dashed curves correspond to

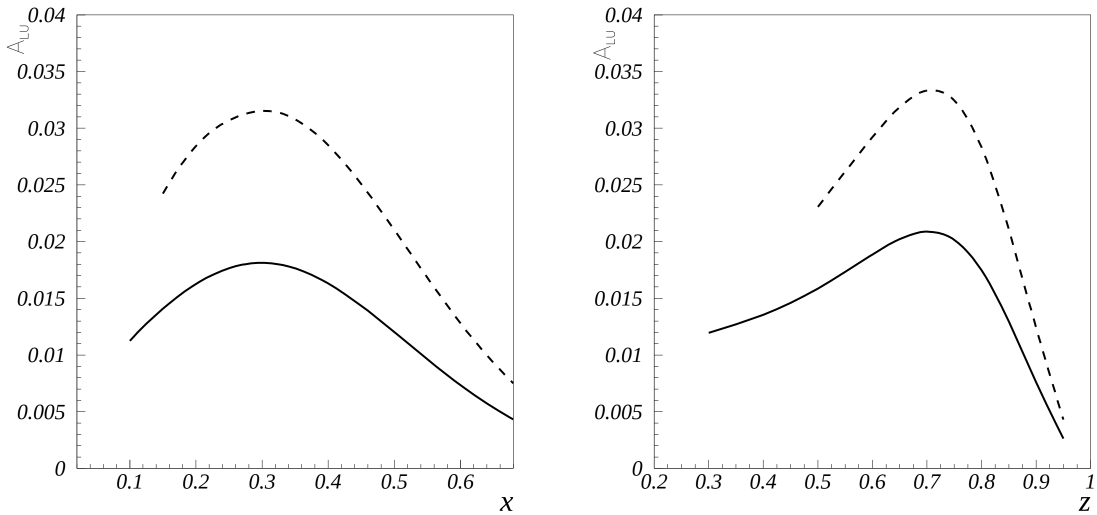

Fig. 3. A_{LU} for π^+ production as a function of x and z at 6 GeV and 12 GeV energies. The dashed curve corresponds to 6 GeV and the full curve to 12 GeV energies.

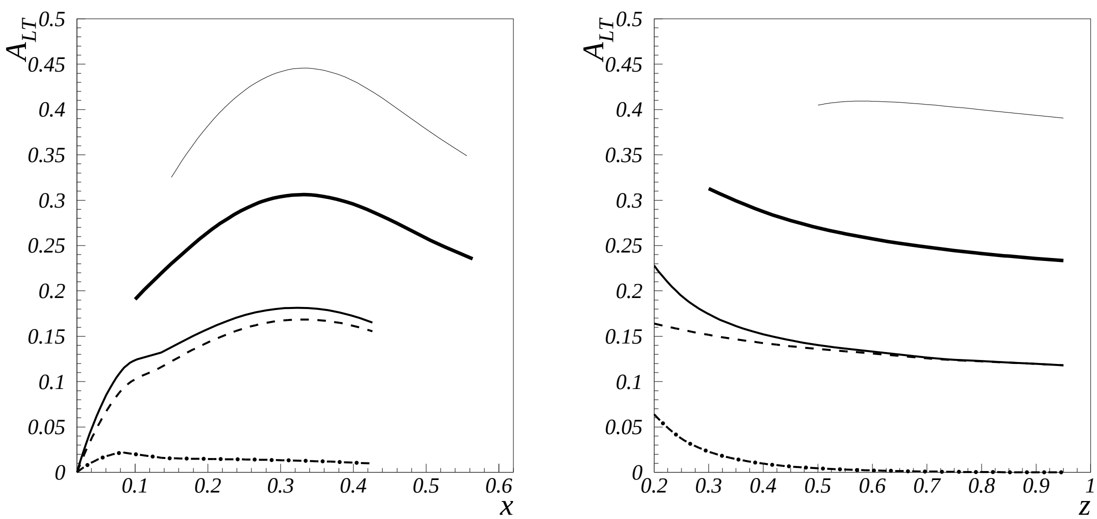

Fig. 4. A_{LT} for π^+ production as a function of x and z at 27.5 GeV energy. The dashed and dot-dashed curves correspond to the contributions of the two terms of Eq. (18) respectively, and the full curve is the sum of those two. The thin curve corresponds to 6 GeV and the thick curve to 12 GeV energies respectively.

the contribution of the chiral-even-even and chiral-odd-odd terms of Eq. (18), respectively, and the full curve is the sum of the two. The thin and thick curves correspond to 6 GeV and 12 GeV beam energies, showing the total asymmetry. The contribution of the term responsible for transversity in A_{LT} is suppressed due to the pion mass and the factor $1/z$. From Fig. 4 one can conclude that the isolation of the chiral-odd effect containing information on quarks transversity from the term, $g_T D_1$, would to present a challenging measurement.

3 Conclusion

The double transverse spin asymmetry which proves to be an interesting observable to probe the effects of higher twist in addition to providing a window into the measurement of transversity has been considered in the quark-scalar diquark framework [21,22]. In this connection, we have explored the twist-three chiral-odd pion fragmentation function and subsequently estimated the double-spin asymmetry with longitudinally polarized electrons scattered on transversely polarized nucleons. This asymmetry contains the product of a chirally odd twist-two transversity distribution and a twist-three fragmentation function. At HERMES [5] and ongoing and upgraded JLAB [23] energies this chiral-odd effect is estimated to be fairly small which makes its isolation from the chiral-even mechanism challenging. In addition the beam spin azimuthal asymmetry, which also contains this sub-leading twist chirally odd fragmentation function, has been calculated for HERMES and JLAB kinematics. It is shown that in the simple quark-diquark model the effects of the twist-three chirally odd fragmentation are suppressed. Consequently, the measurements of BSA can provide valuable information on the leading T-odd fragmentation function, $H_1^{\perp}$, a favored candidate for filtering the transversity properties of the nucleon.

The approach presented in this letter takes into account only up quarks. However the inclusion of axial-vector diquarks may essentially affect the asymmetries [36]. The extension of our results for down quarks and estimates of BSA and double spin asymmetries for π^- and π^0 is a subject of further studies.

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